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C-ideals in ordered power semigroups on semihypergroups induced by Posets*

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Abstract. In this paper, we introduce the concept of C-ideals in the structure called ordered power semigroups on semihypergroups induced by posets which is introduced by authors and study the relationship between the greatest ideal and C-ideals in this structure.

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1. Introduction

The concept of C-ideals in semigroups was investigated by I. Fabrici in 1984 [6]. Later, H. Y. Mao et al. extended the concept of C-ideals in semigroups to ordered semigroups in 2010 [15]. They studied the properties of C-ideals in ordered semigroups. After that, T. Changphas and P. Summaprab also studied the structure of ordered semigroups containing C-ideals in 2016 [1]. Next, Z. Gu et al. characterized ordered semigroups containing the greatest ideal and gave the conditions of the greatest ideal being a C-ideal in ordered semigroups in 2020 [7]. They also introduced the concept of a basis of an ordered semigroup and studied the relationship between the greatest C-ideal and the basis in an ordered semigroup. In this paper, we extend the concept of C-ideals in ordered semigroups to ordered

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power semigroups on semihypergroups induced by posets and study the properties of C-ideals in this structure.

Definition 1.1. Let H be a nonempty set and $P^*(H)$ be the family of all nonempty subsets of H. A mapping $\circ : H \times H \to P^*(H)$ is called a **hyperoperation** on H and $(H, \circ)$ is called a **hypergroupoid**. For any hypergroupoid $(H, \circ)$, S. Ma et al. [**?**] defined the operation $\star : P^*(H) \times P^*(H) \to P^*(H)$ as follows. For any $X, Y \in P^*(H)$, $X \star Y = \bigcup x \circ y$ for all $x \in X, y \in Y$. $\{x\} \star Y$ and $X \star \{y\}$ are denoted by $x \star Y$ and $X \star y$, respectively. Especially, when $X = \{x\}$ and $Y = \{y\}$, $X \star Y = \{x\} \star \{y\} = x \star y$. The algebraic hyperstructure $(H, \circ, \star)$ is called a **semihypergroup** if for every $x, y, z \in H$, $(x \circ y) \star z = x \star (y \circ z)$, i.e. $\bigcup u \circ z = \bigcup x \circ v$ for all $u \in x \circ y, v \in y \circ z$.

Definition 1.2. Let $(H, \circ, \star)$ be a semihypergroup and $\emptyset \neq \mathcal{S} \subseteq P^*(H)$. The operation $\star$ is defined as in Definition **??**. If $\mathcal{S}$ is closed under the operation $\star$ restricted to $\mathcal{S}$ then $(\mathcal{S}, \star)$ is called a **power semigroup on semihypergroup** $(H, \circ, \star)$. We can see that $(\mathcal{S}, \star)$ is also a semigroup. For any nonempty subsets $\mathcal{A}$ and $\mathcal{B}$ of $\mathcal{S}$, we denote $\mathcal{A} \star \mathcal{B} = \{X \star Y \mid X \in \mathcal{A}, Y \in \mathcal{B}\}$.

2. Ordered Power Semigroups on Semihypergroups

The notion of structures on the power set was introduced by M. Szymanska and D. Schweigert in 2001 [17]. The ordered power sets are generalizations of boolean algebras. They defined the relation $\leq_p$ by injective monotone maps as follows.

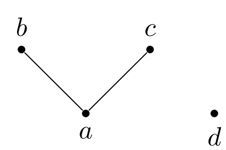
Let $(E, \leq)$ be a finite poset and $P^*(E)$ be the family of all nonempty subsets of E. The relation $\leq_p$ is defined on the power set $P^*(E)$ as follows. For any subsets $\{a_1, \ldots, a_n\}$, $\{b_1, \ldots, b_m\} \subseteq P^*(E)$, $\{a_1, \ldots, a_n\} \leq_p \{b_1, \ldots, b_m\}$ if and only if there exists an injective mapping $\pi : \{a_1, \ldots, a_n\} \to \{b_1, \ldots, b_m\}$ such that $a_i \leq \pi(a_i)$ for $i = 1, \ldots, n$. They also generalized the concept of ordered power sets from the finite set to the general case. For any subsets $\{a_i \mid i \in I\}$, $\{b_j \mid j \in J\} \subseteq P^*(E)$, we have $\{a_i \mid i \in I\} \leq_p \{b_j \mid j \in J\}$ if and only if there exists an injective mapping $\pi : \{a_i \mid i \in I\} \to \{b_j \mid j \in J\}$ such that $a_i \leq \pi(a_i)$ for $i \in I$ and $\{\pi(a_i) \mid i \in I\} \subseteq \{b_j \mid j \in J\}$. Then they proved that the relation $\leq_p$ is antisymmetric. We can see that the relation $\leq_p$ is also reflexive and transitive. That means, it is a partial order. Then $(P^*(E), \leq_p)$ is a partially ordered set which is called an **ordered power set** [17].

We combine the previous notions altogether to construct a new algebraic structure and investigate some properties of C-ideals in this structure.

Definition 2.1. Let $(H, \leq)$ be a poset and $(\mathcal{S}, \star)$ be a power semigroup on semihypergroup $(H, \circ, \star)$. If the relation $\leq_p$ is compatible with the operation $\star$ restricted to $\mathcal{S}$, i.e. for all $X, Y, Z \in \mathcal{S}$, $X \leq_p Y$ implies $Z \star X \leq_p Z \star Y$ and $X \star Z \leq_p Y \star Z$, then we call $(\mathcal{S}, \star, \leq_p)$ an ordered power semigroup on a semihypergroup $(H, \circ, \star)$ induced by a poset $(H, \leq)$.

For any $\emptyset \neq \mathcal{A} \subseteq \mathcal{S}$ where $(\mathcal{S}, \star, \leq_p)$ is an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$, we denote $(\mathcal{A}]_p = \{X \in \mathcal{S} \mid X \leq_p Y$ for some $Y \in \mathcal{A}\}$. If $\mathcal{A} = \{X\}$ then we denote $(\mathcal{A}]_p$ by $(X]_p$.

Example 2.2. Let $H = \{a, b, c, d\}$ and $\leq = \{(a, a), (b, b), (c, c), (d, d), (a, b), (a, c)\}$ be a binary relation on H. It is easily see that $(H, \leq)$ is a poset as the Hasse's diagram.



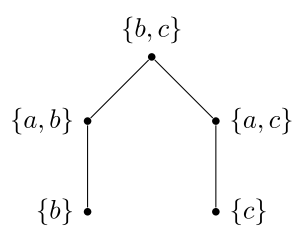
$\circ$	a	b	c	d
a	$\{a\}$	$\{a,c\}$	$\{c\}$	$\{d\}$
b	$\{a,c\}$	$\{a,c\}$	$\{c\}$	$\{d\}$
c	$\{c\}$	$\{c\}$	$\{c\}$	$\{d\}$
d	$\{d\}$	$\{d\}$	$\{d\}$	$\{d\}$

$\star$	$\{c\}$	$\{b\}$	$\{a,b\}$	$\{a,c\}$	$\{b,c\}$
$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$	$\{c\}$
$\{b\}$	$\{c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$
$\{a,b\}$	$\{c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$
$\{a,c\}$	$\{c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$
$\{b,c\}$	$\{c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$	$\{a,c\}$

Define the hyperoperation $\circ$ on H as follows.

The operation $\star$ on $P^*(H)$ is defined as in Definition 1.1. We can see that $(H,\circ,\star)$ is a semihypergroup. Let $\mathcal{S} = \{\{c\},\{b\},\{a,b\},\{a,c\},\{b,c\}\}$. We have the Cayley's table of the operation $\star$ on $\mathcal{S}$ as follows.

Then $(\mathcal{S},\star)$ is a power semigroup on a semihypergroup $(H,\circ,\star)$ and $(\mathcal{S},\star,\leq_p$) can be shown as the following Hasse's diagram.



We can see that the partial order $\leq_p$ is compatible with the operation $\star$ restricted to $\mathcal{S}$. Then $(\mathcal{S},\star,\leq_p)$ is an ordered power semigroup on a semihypergroup $(H,\circ,\star)$ induced by a poset $(H,\leq)$. Let $\mathcal{A} = \{\{c\},\{a,b\}\}$ be a subset of $\mathcal{S}$. Then we have $(\mathcal{A}]_p = \{X \in \mathcal{S} \mid X \leq_p Y$ for some $Y \in \mathcal{A}\} = \{\{b\},\{c\},\{a,b\}\}$.

Lemma 2.2. *Let $(\mathcal{S},\star,\leq_p)$ be an ordered power semigroup on semihypergroup $(H,\circ,\star)$ induced by poset $(H,\leq)$ and $\emptyset \neq \mathcal{A},\mathcal{B} \subseteq \mathcal{S}$. Then the fol-*

lowing statements hold.

(i) $\mathcal{A} \subseteq (\mathcal{A}]_p$*;*

(ii) *If* $\mathcal{A} \subseteq \mathcal{B}$ then $(\mathcal{A}]_p \subseteq (\mathcal{B}]_p$*;*

(iii) $(\mathcal{A}]_p \star (\mathcal{B}]_p \subseteq (\mathcal{A} \star \mathcal{B}]_p$*;*

(iv) $((\mathcal{A}]_p]_p = (\mathcal{A}]_p$*;*

(v) $(\mathcal{A} \cup \mathcal{B}]_p = (\mathcal{A}]_p \cup (\mathcal{B}]_p$*;*

(vi) $(\mathcal{A} \cap \mathcal{B}]_p \subseteq (\mathcal{A}]_p \cap (\mathcal{B}]_p$*; (vii)* $((\mathcal{A}]_p \star (\mathcal{B}]_p]_p = (\mathcal{A} \star \mathcal{B}]_p$.

Definition 2.4. Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. A nonempty subset $\mathcal{A}$ of $\mathcal{S}$ is called a **left (resp. right) ideal of** $\mathcal{S}$ if and only if *(i)* $\mathcal{S} \star \mathcal{A} \subseteq \mathcal{A}$ (resp. $\mathcal{A} \star \mathcal{S} \subseteq \mathcal{A}$)*; (ii)* For $X \in \mathcal{A}$ and $Y \in \mathcal{S}$, $Y \leq_p X$ implies $Y \in \mathcal{A}$. That is $\mathcal{A} = (\mathcal{A}]_p$.

A nonempty subset $\mathcal{A}$ of $\mathcal{S}$ is called an **(two-sided) ideal of** $\mathcal{S}$ if $\mathcal{A}$ is both a left and a right ideal of S. An ideal $\mathcal{A}$ of $\mathcal{S}$ is called a **proper ideal if** $\mathcal{A} \neq \mathcal{S}$. A proper ideal $\mathcal{A}$ of $\mathcal{S}$ is called **the greatest ideal** if every proper ideal is contained in $\mathcal{A}$. A proper ideal $\mathcal{A}$ of $\mathcal{S}$ is called a **maximal ideal** if whenever there exists an ideal $\mathcal{B}$ of $\mathcal{S}$ such that $\mathcal{A} \subseteq \mathcal{B}$ then $\mathcal{B} = \mathcal{S}$. If $\mathcal{S}$ contains no proper ideals then $\mathcal{S}$ is called **simple**.

Lemma 2.5. *Let* $(\mathcal{S}, \star, \leq_p)$ *be an ordered power semigroup on semihypergroup* $(H, \circ, \star)$ *induced by poset* $(H, \leq)$*. Then the following statements hold.*

(i) *If* $\mathcal{A}, \mathcal{B}$ are ideals of $\mathcal{S}$ then $(\mathcal{A} \star \mathcal{B}]_p$ is an ideal of $\mathcal{S}$*;*

(ii) *If* $\mathcal{A}_1, ..., \mathcal{A}_n$ are ideals of $\mathcal{S}$ for $n \in \mathbb{N}$ then $\mathcal{A}_1 \star \cdots \star \mathcal{A}_n \subseteq \mathcal{A}_1 \cap ... \cap \mathcal{A}_n$*;*

(iii) *Union of ideals of* $\mathcal{S}$ *is an ideal of* $\mathcal{S}$;

(iv) *Finite intersection of ideals of $\mathcal{S}$ is an ideal of $\mathcal{S}$;*

(v) *If $\mathcal{A} \subseteq \mathcal{S}$ then $(\mathcal{S} \star \mathcal{A} \star \mathcal{S}]_p$ is an ideal of $\mathcal{S}$.*

Proof.

(i) Let $K \in (\mathcal{A} \star \mathcal{B}]_p$ and $W \in \mathcal{S}$. Then there exists $K_1 \in \mathcal{A} \star \mathcal{B}$ such that $K \leq_p K_1$. Then $K_1 = K_2 \star K_3$ for some $K_2 \in \mathcal{A}$ and $K_3 \in \mathcal{B}$. That is, $W \star K \leq_p W \star K_1 = W \star K_2 \star K_3 \in \mathcal{S} \star \mathcal{A} \star \mathcal{B} \subseteq \mathcal{A} \star \mathcal{B}$ and $K \star W \leq_p K_1 \star W = K_2 \star K_3 \star W \in \mathcal{A} \star \mathcal{B} \star \mathcal{S} \subseteq \mathcal{A} \star \mathcal{B}$. Hence $W \star K, K \star W \in (\mathcal{A} \star \mathcal{B}]_p$. Let $X \in \mathcal{S}$, $Y \in (\mathcal{A} \star \mathcal{B}]_p$ and $X \leq_p Y$. Then $X \in ((\mathcal{A} \star \mathcal{B}]_p]_p = (\mathcal{A} \star \mathcal{B}]_p$. Therefore $(\mathcal{A} \star \mathcal{B}]_p$ is an ideal of $\mathcal{S}$.

(ii) Let $\mathcal{A}_1, ..., \mathcal{A}_n$ are ideals of $\mathcal{S}$ for $n \in \mathbb{N}$. Let $X \in \mathcal{A}_1 \star \cdots \star \mathcal{A}_n$. Then $X = X_1 \star ... \star X_n$ for some $X_i \in \mathcal{A}_i$ and $i \in \{1, \cdots, n\}$. Since $X_1 \in \mathcal{A}_1$, $X_2 \star \cdots \star X_n \in \mathcal{A}_2 \star ... \star \mathcal{A}_n \subseteq \mathcal{S}$ and $\mathcal{A}_1$ is an ideal of $\mathcal{S}$, we have $X_1 \star \cdots \star X_n \in \mathcal{A}_1$. Since $X_1 \star ... \star X_{n-1} \in \mathcal{A}_1 \star \cdots \star \mathcal{A}_{n-1} \subseteq \mathcal{S}$, $X_n \in \mathcal{A}_n$ and $\mathcal{A}_n$ is an ideal of $\mathcal{S}$, we have $X_1 \star \cdots \star X_n \in \mathcal{A}_n$. Since $X_j \in \mathcal{A}_j$, $X_1 \star \cdots \star X_{j-1}, X_{j+1} \star \cdots \star X_n \in \mathcal{S}$ for some $j \in \{2, \cdots, n-1\}$ and $\mathcal{A}_j$ is an ideal of $\mathcal{S}$, we have $X_1 \star \cdots \star X_{j-1} \star X_j \star X_{j+1} \star \cdots \star X_n \in \mathcal{A}_j$.

That is, $X = X_1 \star \cdots \star X_n \in \mathcal{A}_1 \cap ... \cap \mathcal{A}_n$. Therefore $\mathcal{A}_1 \star \cdots \star \mathcal{A}_n \subseteq \mathcal{A}_1 \cap \cdots \cap \mathcal{A}_n$.

(iii) Let $\{\mathcal{A}_i \mid i \in I\}$ be a family of ideals of $\mathcal{S}$. Then $\emptyset \neq \bigcup_{i \in I} \mathcal{A}_i \subseteq \mathcal{S}$. Let $K \in \bigcup_{i \in I} \mathcal{A}_i$ and $W \in \mathcal{S}$. Then $K \in \mathcal{A}_j$ for some $j \in I$. Hence $K \star W \in \mathcal{A}_j$ and $W \star K \in \mathcal{A}_j$ for some $j \in I$. Then $K \star W \in \bigcup_{i \in I} \mathcal{A}_i$ and $W \star K \in \bigcup_{i \in I} \mathcal{A}_i$. Let $X \in \mathcal{S}$, $Y \in \bigcup_{i \in I} \mathcal{A}_i$ and $X \leq_p Y$. Since $Y \in \mathcal{A}_j$ for some $j \in I$ and $\mathcal{A}_j$ is an ideal of $\mathcal{S}$, we have $X \in \mathcal{A}_j$. That is, $X \in \bigcup_{i \in I} \mathcal{A}_i$. Therefore $\bigcup_{i \in I} \mathcal{A}_i$ is an ideal of $\mathcal{S}$.

(iv) Let $\{\mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_n\}$ be a family of ideals of $\mathcal{S}$. Then by (ii), $\emptyset \neq \bigcap_{i=1}^{n} \mathcal{A}_i \subseteq \mathcal{S}$ for all $i \in \{1 \cdots, n\}$. Let $K \in \bigcap_{i=1}^{n} \mathcal{A}_i$ and $W \in \mathcal{S}$.

Then $K \in \mathcal{A}_j$ for all $j \in \{1, \cdots, n\}$.

Hence $K \star W \in \mathcal{A}_j$ and $W \star K \in \mathcal{A}_j$ for all $j \in \{1, \cdots, n\}$. Then $K \star W \in \bigcap_{i=1}^{n} \mathcal{A}_i$ and $W \star K \in \bigcap_{i=1}^{n} \mathcal{A}_i$. Let $X \in \mathcal{S}$, $Y \in \bigcap_{i=1}^{n} \mathcal{A}_i$ and $X \leq_p Y$. Since $Y \in \mathcal{A}_j$ for all $j \in \{1, \cdots, n\}$ and $\mathcal{A}_j$ is an ideal of $\mathcal{S}$, we have $X \in \mathcal{A}_j$ for all $j \in \{1, ..., n\}$. That is, $X \in \bigcap_{i=1}^{n} \mathcal{A}_i$. Therefore $\bigcap_{i=1}^{n} \mathcal{A}_i$ is an ideal of $\mathcal{S}$.

(v) Let $\mathcal{A} \subseteq \mathcal{S}$ and $K \in (\mathcal{S} \star \mathcal{A} \star \mathcal{S}]_p$ and $W \in \mathcal{S}$. Then there exists $K_1 \in \mathcal{S} \star \mathcal{A} \star \mathcal{S}$ such that $K \leq_p K_1$. Then $K_1 = K_2 \star X \star K_3$ for some $K_2, K_3 \in \mathcal{S}$ and $X \in \mathcal{A}$. Hence $W \star K \leq_p W \star K_1 = W \star K_2 \star X \star K_3 \in \mathcal{S} \star \mathcal{S} \star \mathcal{A} \star \mathcal{S} \subseteq \mathcal{S} \star \mathcal{A} \star \mathcal{S}$ and $K_1 \star W \leq_p K_2 \star X \star K_3 \star W \in \mathcal{S} \star \mathcal{A} \star \mathcal{S} \star \mathcal{S} \subseteq \mathcal{S} \star \mathcal{A} \star \mathcal{S}$. Let $X \in \mathcal{S}$, $Y \in (\mathcal{S} \star \mathcal{A} \star \mathcal{S}]_p$ and $X \leq_p Y$. Then $X \in ((\mathcal{S} \star \mathcal{A} \star \mathcal{S}]_p]_p = (\mathcal{S} \star \mathcal{A} \star \mathcal{S}]_p$. Therefore $(\mathcal{S} \star \mathcal{A} \star \mathcal{S}]_p$ is an ideal of $\mathcal{S}$.

□

Lemma 2.6. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$ and $\emptyset \neq \mathcal{A} \subseteq \mathcal{S}$. Then $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$ (resp. $(\mathcal{A} \cup \mathcal{A} \star \mathcal{S}]_p$, $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A} \cup \mathcal{A} \star \mathcal{S} \cup \mathcal{S} \star \mathcal{A} \star \mathcal{S}]_p$) is the smallest left (resp. right, two-sided) ideal of $\mathcal{S}$ containing $\mathcal{A}$.*

Proof. First, we will show that $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$ is a left ideal of $\mathcal{S}$ containing $\mathcal{A}$. We have $\emptyset \neq \mathcal{A} \subseteq \mathcal{A} \cup \mathcal{S} \star \mathcal{A} \subseteq (\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p \subseteq \mathcal{S}$. Then $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$ is not an empty set. Let $K \in \mathcal{S}$, $W \in (\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$. Then there exists $W_1 \in \mathcal{A} \cup \mathcal{S} \star \mathcal{A}$ such that $W \leq_p W_1$. Then $W_1 \in \mathcal{A}$ or $W_1 \in \mathcal{S} \star \mathcal{A}$.

Assume that $W_1 \notin \mathcal{A}$. We have $W_1 \in \mathcal{S} \star \mathcal{A}$. Then $K \star W \leq_p K \star W_1 \in \mathcal{S} \star \mathcal{S} \star \mathcal{A} \subseteq \mathcal{S} \star \mathcal{A} \subseteq \mathcal{A} \cup \mathcal{S} \star \mathcal{A}$. That is, $K \star W \in (\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$. Let $K_1 \in \mathcal{S}$, $K_2 \in (\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$ and $K_1 \leq_p K_2$. Then $K_1 \in ((\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p]_p = (\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$. Hence $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$ is a left ideal of $\mathcal{S}$ containing $\mathcal{A}$. Next, we will show that $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$ is the smallest left ideal of $\mathcal{S}$ containing $\mathcal{A}$. Let $\mathcal{B}$ be a left ideal of

$\mathcal{S}$ containing $\mathcal{A}$. Since $\mathcal{A} \subseteq \mathcal{B}$, we can see that $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p \subseteq (\mathcal{B} \cup \mathcal{S} \star \mathcal{B}]_p = \mathcal{B}$. That is, $(\mathcal{A} \cup \mathcal{S} \star \mathcal{A}]_p$ is the smallest left ideal of $\mathcal{S}$ containing $\mathcal{A}$. □

3. C-ideals in ordered power semigroups on semihypergroups induced by Posets

Next, we introduce the concept of C-ideals in ordered power semigroups on semihypergroups induced by posets and study its algebraic properties. We call $(\mathcal{S}, \star, \leq_p)$ an **ordered power semigroup on semihypergroup induced by poset** H **with an identity** 1 if there exists $1 \in \mathcal{S}$ such that $X \star 1 = 1 \star X = X$ for all $X \in \mathcal{S}$. 1 is called an **identity element of** $\mathcal{S}$.

Definition 3.1. Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. A proper ideal $\mathcal{A}$ of $\mathcal{S}$ is called a **covered ideal (C-ideal)** of $\mathcal{S}$ if $\mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$.

Lemma 3.2. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$ and $\mathcal{A}$ be a proper ideal of $\mathcal{S}$. Then $(\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$ is an ideal of $\mathcal{S}$.*

Proof. Let $\mathcal{A}$ be a proper ideal of $\mathcal{S}$. We will show that $\mathcal{S} \star (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$ and $(\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \star \mathcal{S} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Let $X \in \mathcal{S} \star (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Then $X = X_1 \star X_2$ such that $X_1 \in \mathcal{S}$ and $X_2 \in (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Since $X_2 \in (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$, there are $Y \in \mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}$ such that $X_2 \leq_p Y$. Then $X_1 \star X_2 \leq_p X_1 \star Y$.

Consider $X_1 \star Y \in \mathcal{S} \star \mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S} \subseteq \mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}$. Then $X = X_1 \star X_2 \in (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Therefore $\mathcal{S} \star (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Similarly, $(\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \star \mathcal{S} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Let $Z_1 \in \mathcal{S}$ and $Z_2 \in (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$ such that $Z_1 \leq_p Z_2$. Then $Z_1 \in ((\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p]_p = (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Therefore $(\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$

is an ideal of $\mathcal{S}$. □

Theorem 3.3. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{S}$ is not simple then $\mathcal{S}$ contains at least one C-ideal.*

Proof. Let $\mathcal{S}$ be not simple. There exists an ideal $\mathcal{A}$ of $\mathcal{S}$ such that $\mathcal{A}$ is a proper ideal. Then $\mathcal{S} - \mathcal{A} \neq \emptyset$. Since $(\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$ is an ideal of $\mathcal{S}$, we have $\mathcal{B} = \mathcal{A} \cap (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$ is an ideal of $\mathcal{S}$. Consider $\mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$ and $(\mathcal{S} - \mathcal{A}) \subseteq (\mathcal{S} - \mathcal{B})$. Then $\mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$. Therefore $\mathcal{B}$ is a C-ideal of $\mathcal{S}$. □

Theorem 3.4. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ and $\mathcal{B}$ are different proper ideals of $\mathcal{S}$ such that $\mathcal{A} \cup \mathcal{B} = \mathcal{S}$ then $\mathcal{A}$ and $\mathcal{B}$ are not C-ideals of $\mathcal{S}$.*

Proof. Let $\mathcal{A}$ and $\mathcal{B}$ be two different proper ideals of $\mathcal{S}$ such that $\mathcal{A} \cup \mathcal{B} = \mathcal{S}$. Suppose that $\mathcal{A}$ is a C-ideal of $\mathcal{S}$. We have $\mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Since $\mathcal{A} \cup \mathcal{B} = \mathcal{S}$, we have $\mathcal{S} - \mathcal{A} \subseteq \mathcal{B}$. Then $\mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \subseteq (\mathcal{S} \star \mathcal{B} \star \mathcal{S}]_p \subseteq (\mathcal{B}]_p = \mathcal{B}$. Since $\mathcal{A} \cup \mathcal{B} = \mathcal{S}$ and $\mathcal{A} \subseteq \mathcal{B}$, we have $\mathcal{B} = \mathcal{S}$. This is a contradiction with $\mathcal{B}$ is a proper ideal of $\mathcal{S}$. Then $\mathcal{A}$ and $\mathcal{B}$ are not C-ideals of $\mathcal{S}$. □

Corollary 3.5. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{S}$ contains more than one maximal ideal then none of them is a C-ideal of $\mathcal{S}$.*

Proof. Let $\mathcal{A}$ and $\mathcal{B}$ be two different maximal ideals of $\mathcal{S}$. Then $\mathcal{A}$ and $\mathcal{B}$ are two different proper ideals. Since $\mathcal{A} \subset \mathcal{A} \cup \mathcal{B}$, $\mathcal{A} \cup \mathcal{B}$ is an ideal of $\mathcal{S}$ and $\mathcal{A}$ is a maximal ideal of $\mathcal{S}$, we have $\mathcal{A} \cup \mathcal{B} = \mathcal{S}$. By Theorem **??**, $\mathcal{A}$ and $\mathcal{B}$ are not C-ideals of $\mathcal{S}$. □

Theorem 3.6. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihy-*

pergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ is only one maximal ideal of $\mathcal{S}$ and $\mathcal{A}$ is a C-ideal then $\mathcal{A}$ is the greatest ideal of $\mathcal{S}$.

Proof. Let $\mathcal{A}$ be only one maximal ideal of $\mathcal{S}$ and $\mathcal{A}$ is a C-ideal. Suppose that $\mathcal{B}$ is a proper ideal of $\mathcal{S}$. Since $\mathcal{A}$ is a C-ideal, we have $\mathcal{A} \cup \mathcal{B} \neq \mathcal{S}$ by Theorem 3.7. Since $\mathcal{A} \subseteq \mathcal{A} \cup \mathcal{B} \neq \mathcal{S}$ and $\mathcal{A}$ is maximal ideal of $\mathcal{S}$, we have $\mathcal{A} = \mathcal{A} \cup \mathcal{B}$. Then $\mathcal{B} \subseteq \mathcal{A}$. Therefore $\mathcal{A}$ is the greatest ideal of $\mathcal{S}$. $\square$

Theorem 3.7. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ is only one maximal ideal of $\mathcal{S}$ then $\mathcal{A}$ is the greatest ideal of $\mathcal{S}$.*

Proof. Let $\mathcal{A}$ be only one maximal ideal of $\mathcal{S}$. Suppose that $\mathcal{B}$ is a proper ideal of $\mathcal{S}$. Since $\mathcal{A}$ is only one maximal ideal of $\mathcal{S}$, we have $\mathcal{B} \subseteq \mathcal{A}$. Therefore $\mathcal{A}$ is the greatest ideal of $\mathcal{S}$. $\square$

Theorem 3.8. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ and $\mathcal{B}$ are C-ideals of $\mathcal{S}$ then $\mathcal{A} \cup \mathcal{B}$ is a C-ideal of $\mathcal{S}$.*

Proof. Let $\mathcal{A}$ and $\mathcal{B}$ be C-ideals of $\mathcal{S}$. Then $\mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$ and $\mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$. Let $X \in \mathcal{A} \cup \mathcal{B}$. Then $X \in \mathcal{A}$ or $X \in \mathcal{B}$. Suppose that $X \in \mathcal{A}$. Since $\mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$, we have $X \in (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. There exists $Y \in \mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}$ such that $X \leq_p Y$. Then $Y = Y_1 \star Z \star Y_2$ for some $Y_1, Y_2 \in \mathcal{S}$ and $Z \in \mathcal{S} - \mathcal{A}$. Since $\mathcal{A} \subseteq \mathcal{A} \cup \mathcal{B}$, we have $\mathcal{S} - (\mathcal{A} \cup \mathcal{B}) \subseteq \mathcal{S} - \mathcal{A}$. If $Z \in \mathcal{S} - (\mathcal{A} \cup \mathcal{B})$. Then $Y = Y_1 \star Z \star Y_2 \in \mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cup \mathcal{B})) \star \mathcal{S}$. Since $X \leq_p Y$, we have $X \in (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cup \mathcal{B})) \star \mathcal{S}]_p$. Hence $\mathcal{A} \cup \mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cup \mathcal{B})) \star \mathcal{S}]_p$.

If $Z \in \mathcal{A} \cup \mathcal{B}$. Since $Z \in \mathcal{S} - \mathcal{A}$, we have $Z \in \mathcal{B}$. Since $\mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$, we have $Z \in (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$. Then $\mathcal{S} - \mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$ and there exists $W \in \mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}$ such that $Z \leq_p W$. Then $W = W_1 \star V \star W_2$ for some $W_1, W_2 \in \mathcal{S}$ and $V \in \mathcal{S} - \mathcal{B}$. Since $X \leq_p Y$, we have $X \leq_p Y =$

$Y_1 \star Z \star Y_2 \leq_p Y_1 \star W \star Y_2 = Y_1 \star (W_1 \star V \star W_2) \star Y_2 = (Y_1 \star W_1) \star V \star (W_2 \star Y_2)$.

Consider $V \in \mathcal{S} - \mathcal{B}$ and $\mathcal{S} - (\mathcal{A} \cup \mathcal{B}) \subseteq \mathcal{S} - \mathcal{B}$. If $V \in \mathcal{A} \cup \mathcal{B}$. Since $V \in \mathcal{S} - \mathcal{B}$, we have $V \in \mathcal{A}$. Since $\mathcal{A}$ is an ideals of $\mathcal{S}$, we have $Z \leq_p W = W_1 \star V \star W_2 \in \mathcal{S} \star \mathcal{A} \star \mathcal{S} \subseteq \mathcal{A}$. Then $Z \in (\mathcal{A}]_p = \mathcal{A}$. This is a contradiction with $Z \in \mathcal{S} - \mathcal{A}$. Thus $V \in \mathcal{S} - (\mathcal{A} \cup \mathcal{B})$. It implies that $(Y_1 \star W_1) \star V \star (W_2 \star Y_2) \in \mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cup \mathcal{B})) \star \mathcal{S}$. Then $X \in (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cup \mathcal{B})) \star \mathcal{S}]_p$. Hence $\mathcal{A} \cup \mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cup \mathcal{B})) \star \mathcal{S}]_p$.

Similarly with $X \in \mathcal{B}$, we have $\mathcal{A} \cup \mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cup \mathcal{B})) \star \mathcal{S}]_p$. Therefore $\mathcal{A} \cup \mathcal{B}$ is a C-ideal of $\mathcal{S}$. □

Theorem 3.9. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ is an ideal of $\mathcal{S}$ and $\mathcal{B}$ is a C-ideal of $\mathcal{S}$ then $\mathcal{A} \cap \mathcal{B}$ is a C-ideal of $\mathcal{S}$.*

Proof. Let $\mathcal{A}$ be an ideal of $\mathcal{S}$ and $\mathcal{B}$ be a C-ideal of $\mathcal{S}$. Then $\mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$. Let $X \in \mathcal{A} \cap \mathcal{B}$. Then $X \in \mathcal{A}$ and $X \in \mathcal{B}$. Since $\mathcal{S} - \mathcal{B} \subseteq \mathcal{S} - (\mathcal{A} \cap \mathcal{B})$, we have $(\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p \subseteq (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cap \mathcal{B})) \star \mathcal{S}]_p$. Then $X \in (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cap \mathcal{B})) \star \mathcal{S}]_p$. Hence $\mathcal{A} \cap \mathcal{B} \subseteq (\mathcal{S} \star (\mathcal{S} - (\mathcal{A} \cap \mathcal{B})) \star \mathcal{S}]_p$. Therefore $\mathcal{A} \cap \mathcal{B}$ is a C-ideal of $\mathcal{S}$. □

Corollary 3.10. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ and $\mathcal{B}$ are C-ideals of $\mathcal{S}$ then $\mathcal{A} \cap \mathcal{B}$ is a C-ideal of $\mathcal{S}$.*

Theorem 3.11. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ is both the greatest ideal and a C-ideal of $\mathcal{S}$ then every proper ideal of $\mathcal{S}$ is a C-ideal.*

Proof. Let $\mathcal{A}$ be both the greatest ideal and a C-ideal of $\mathcal{S}$. Then $\mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Let $\mathcal{B}$ be a proper ideal of $\mathcal{S}$. Then $\mathcal{B} \subseteq \mathcal{A}$. Hence $(\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$.

Then $\mathcal{B} \subseteq \mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{B}) \star \mathcal{S}]_p$. Therefore $\mathcal{B}$ is a C-ideal of $\mathcal{S}$. □

Theorem 3.12. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{S}$ has an identity then every proper ideal of $\mathcal{S}$ is a C-ideal.*

Proof. Let 1 be an identity of $\mathcal{S}$ and $\mathcal{A}$ be a proper ideal of $\mathcal{S}$. Suppose that $1 \in \mathcal{A}$. Then $\mathcal{S} = \mathcal{S} \star 1 \subseteq \mathcal{S} \star \mathcal{A} = \mathcal{A}$. This is a contradiction with $\mathcal{A}$ is a proper ideal. Then $1 \notin \mathcal{A}$. That is, $1 \in \mathcal{S} - \mathcal{A}$. Then $\mathcal{S} = \mathcal{S} \star 1 \star 1 \subseteq \mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Hence $(\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p = \mathcal{S}$. Then $\mathcal{A} \subseteq (\mathcal{S} \star (\mathcal{S} - \mathcal{A}) \star \mathcal{S}]_p$. Therefore $\mathcal{A}$ is a C-ideal of $\mathcal{S}$. □

Corollary 3.13. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ is the greatest ideal of $\mathcal{S}$ with an identity then $\mathcal{A}$ is a C-ideal.*

In the following, we study ordered power semigroups on semihypergroups induced by posets containing the greatest ideal and give the conditions of the greatest ideal to be a C-ideal in an ordered power semigroup on semihypergroup induced by poset.

For any $X \in \mathcal{S}$, $I(X)$ denotes the principal ideal of $\mathcal{S}$ generated by X. Then we have $I(X) = (X \cup \mathcal{S} \star X \cup X \star \mathcal{S} \cup \mathcal{S} \star X \star \mathcal{S}]_p$ is the smallest ideal of $\mathcal{S}$ containing X. Green's relation J on ordered semigroups was introduced by N. Kehayopulu in [12].

Next, we will study Green's relation J on ordered power semigroup on semihypergroup induced by poset and define the Green's relation J in the usual way as follows. $J = \{(X, Y) \in \mathcal{S} \times \mathcal{S} \mid I(X) = I(Y)\}$. We can see that the relation J is an equivalence relation on $\mathcal{S}$. For any $X \in \mathcal{S}$, we denote the J-class containing X by J_X and we define a relation $\leq_j$ on the set of all J-classes in $\mathcal{S}$ as follows. For any $X, Y \in \mathcal{S}$, $J_X \leq_j J_Y$ if and only if $I(X) \subseteq I(Y)$. We can see that the relation $\leq_j$ is a partial order on $\mathcal{S}$. A J-class J_X of $\mathcal{S}$ is called **maximal** if there is no other J-class J_Y such that $J_X <_j J_Y$. A J-class J_X of $\mathcal{S}$ is called **the greatest J-class** if other J-classes are all contained in J_X.

Lemma 3.14. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$ and $\mathcal{A}$ be the greatest ideal of $\mathcal{S}$. Then $I(X) = \mathcal{S}$ if and only if $X \in \mathcal{S} - \mathcal{A}$.*

Proof. Let $\mathcal{A}$ be the greatest ideal of $\mathcal{S}$. Then $\mathcal{A}$ is a proper ideal of $\mathcal{S}$.

($\Longrightarrow$) Let $I(X) = \mathcal{S}$. Suppose $X \in \mathcal{A}$. Then $\mathcal{A}$ is an ideal of $\mathcal{S}$ containing X. Since $I(X)$ is the smallest ideal of $\mathcal{S}$ containing X, we have $I(X) \subseteq \mathcal{A}$. Since $I(X) = \mathcal{S}$, we have $\mathcal{A} = \mathcal{S}$. This is a contradiction with $\mathcal{A}$ is a proper ideal of $\mathcal{S}$. Then $X \in \mathcal{S} - \mathcal{A}$.

($\Longleftarrow$) Let $X \in \mathcal{S} - \mathcal{A}$. Suppose $I(X)$ is a proper ideal of $\mathcal{S}$. Since $\mathcal{A}$ is the greatest ideal of $\mathcal{S}$, we have $I(X) \subseteq \mathcal{A}$. Since $X \in I(X)$ and $I(X) \subseteq \mathcal{A}$, we have $X \in \mathcal{A}$. This is a contradiction with $X \in \mathcal{S} - \mathcal{A}$. Then $I(X) = \mathcal{S}$ for any $X \in \mathcal{S} - \mathcal{A}$. $\square$

Lemma 3.15. *Let $(\mathcal{S}, \star, \leq_p)$ be an ordered power semigroup on semihypergroup $(H, \circ, \star)$ induced by poset $(H, \leq)$. If $\mathcal{A}$ is the greatest ideal of $\mathcal{S}$ then $\mathcal{S} - \mathcal{A}$ is the greatest J-class of $\mathcal{S}$.*

Proof. Let $\mathcal{A}$ be the greatest ideal of $\mathcal{S}$ and $X \in \mathcal{S} - \mathcal{A}$. Then $\mathcal{A}$ is a proper ideal. By Lemma **??**, we have $I(X) = \mathcal{S}$.

We will show that $\mathcal{S} - \mathcal{A}$ is a J-class containing X. That is $\mathcal{S} - \mathcal{A} = J_X$. Let $U \in J_X$. Then $I(U) = I(X)$. Since $I(X) = \mathcal{S}$, we have $I(U) = I(X) = \mathcal{S}$. By Lemma 3.14, we have $U \in \mathcal{S} - \mathcal{A}$. Hence $J_X \subseteq \mathcal{S} - \mathcal{A}$.

Let $V \in \mathcal{S} - \mathcal{A}$. By Lemma 3.14, we have $I(V) = \mathcal{S} = I(X)$. Then $V \in J_X$. Hence $\mathcal{S} - \mathcal{A} = J_X$. Next, we will show that $\mathcal{S} - \mathcal{A}$ is the greatest J-class of $\mathcal{S}$. Let J_Y be a J-class containing Y and $Y \in \mathcal{S}$. Since $X \in \mathcal{S} - \mathcal{A}$, we have $I(X) = \mathcal{S}$. Since $I(Y) \subseteq \mathcal{S}$ and $I(X) = \mathcal{S}$, we have $I(Y) \subseteq I(X)$. That is $J_Y \leq_j J_X = \mathcal{S} - \mathcal{A}$. Hence $\mathcal{S} - \mathcal{A}$ is the greatest J-class of $\mathcal{S}$. □

4. Conclusion

In this paper, we introduced the concept of C-ideals in ordered power semigroups on semihypergroups induced by posets and gave some of its examples. Furthermore, we studied some of its algebraic properties and characterized C-ideals in ordered power semigroups on semihypergroups induced by posets in various ways. The result is similarly with C-ideals in ordered semigroup but the proof is slightly different.

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